Published by Institute of Physics Publishing for SISSA/ISAS
Received: *May 27, 2002*
Revised: *July 11, 2002*
Accepted: *August 16, 2002*

Matrix strings in pp-wave backgrounds from deformed super Yang-Mills theory

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Abstract: We formulate matrix models for strings in ten dimensional pp-wave backgrounds and for particles in eleven dimensional ones. This is done by first characterizing the deformations of ten dimensional N=1 SYM which are induced by a constant bispinorial coupling $\int \bar{\Psi} H \Psi$ plus a minimal purely bosonic completition and then by the appropriate dimensional reduction. We find a whole class of new models for the matrix strings and a generalization of the supernumerary supersymmetric models as far as the matrix theory for particles is concerned. A companion deformation of the IKKT matrix model is also discussed.

Keywords: M(atrix) Theories, Penrose limit and pp-wave background, Supersymmetry Breaking.

JHEP08(2002)022

Contents

1. Introduction

Despite a lot of efforts, string theory in curved backgrounds remains a hard problem to be understood. This is because, generically, the perturbative methods apply only if the gravitational background is almost flat (in string units) and a true nonperturbative formulation of the theory is still to be given. The only partly positive results in this direction being some cases of homogeneous spaces whose study is available because of the power of the WZNW model (see for example [1]).

A simple case of tractable model is the string in the pp-wave [2]. The pp-wave solution of IIB ten dimensional supergravity is given by the metric

$$ds^2 = -4dx_+dx_- - f^2\mathbf{x}^2 dx^{+2} + d\mathbf{x}^2 \tag{1.1}$$

where $\mathbf{x}$ is the eight dimensional coordinate vector, and the self-dual constant RR 5-form

$$F_{+1234} = F_{+5678} = f \times C$$

where C is a non zero numerical constant. The light-cone GS string obtained in [2] is given by the quadratic action

$$S = \frac{1}{2\pi\alpha'}\int d\xi^2 \left(\frac{1}{2}\partial_+\mathbf{x}\partial_-\mathbf{x} - \frac{1}{2}f^2\mathbf{x}^2 + [\text{fermions}]\right) \tag{1.2}$$

where the fermionic term contains the proper RR-vertex at zero momentum. The simplicity of the above σ-model makes the theory then tractable.

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The additional importance of studying this background is that it can be obtained as a particular contraction of the AdS×S [3, 4] and therefore its analysis is linked to the general issue of holographic gauge/string correspondence. This point has been addressed in [3], where the spectrum of strings in the pp-wave is recontructed from $\mathcal{N} = 4$ SYM in four dimension and is a presently studied subject [5].

In this framework, there appears a strong resemblance in the string spectrum recontruction by the gauge theory operators with the world-sheet generation in Matrix String Theory (MST). This suggests a deeper relation within the holographic approach between the string-bits and the four dimensional gauge theory.

Moreover, the problem of MST in the pp-wave is interesting also by itself, since no curved background has been implemented in the MST framework so far.

A possible strategy to obtain a matrix model for strings on the pp-wave might be to compactify along an isometric direction the eleven dimensional matrix models for pp-waves. This compactification is rather troubling because of the (space-)time dependence of the supersymmetry parameters and therefore the results obtainable this way need to be carefully analized from the point of view of supersymmetry.

An alternative strategy for model building, which will be developed in this letter, is a constructive one. We will first study deformations of ten dimensional super Yang-Mills (SYM) [6] induced by the addition of a quadratic term in the spinorial field – accompanied by a suitable purely bosonic term – in the action and then see under dimensional reduction to two dimensions which kind of deformed matrix string models are available.

As a counter-check we study the deformations of matrix theory induced by this mechanism via dimensional reduction to one dimension and we find the pp-wave martix theory of [7] as a particular solution of more general possible structures.

In the next section we study the possible deformations of N=1 SYM in ten dimensions induced by the addition of a constant bi-spinorial coupling as $\int \bar{\Psi} H \Psi$ (plus purely bosonic terms) with a particular attention to the surviving supersymmetries. In the next section we perform a dimensional reduction from ten to one dimension of the results of the previous section and find the relevant matrix models for particles. In the third section we face the problem of a formulation of matrix string theory on pp-wave background and find the existence of a plethora of new models with supernumerary supersymmetries. The last section is left for the conclusions and some open questions. A deformation of the IKKT matrix model [8] is also discussed in appendix A.

2. Deformation of SYM via a constant bispinor

In this section we describe a general deformation scheme for $\mathcal{N} = 1$ supersymmetric Yang-Mills theories by applying it to the ten dimensional case. This deformation is induced by the addition to the action of a purely quadratic fermionic term plus some bosonic completition. Here we study the general equations for the deformed supersymmetry.

The action of $\mathcal{N} = 1$ D=10 SYM theory with gauge group U(N) is

$$S^0_{10} = \frac{1}{g^2_{10}} \int d^{10}x \, \mathrm{Tr} \left(-\frac{1}{4} F^2 + \frac{i}{2} \Psi^T \Gamma^0 \Gamma^\mu D_\mu \Psi \right) \tag{2.1}$$

where Ψ is a Weyl-Majorana spinor and in the adjoint representation of the $\mathrm{U}(N)$ gauge group. The action is invariant under the following supersymmetries

$$\delta^0 A_\mu = \frac{i}{2}\bar{\epsilon}\Gamma_\mu\Psi \qquad \delta^0\Psi = -\frac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon \tag{2.2}$$

$$\delta^{0'} A_\mu = 0 \qquad \delta^{0'}\Psi = \epsilon' \tag{2.3}$$

where ϵ and ϵ' are *constant* Weyl-Majorana spinors.

Notice that, if we perform a variation with ϵ and ϵ' *non constant* Weyl-Majorana spinors, we get

$$\delta S^0_{10} = \frac{1}{g^2_{10}}\int d^{10}x \,\mathrm{Tr}\,\bar{\Psi}\left\{-\frac{i}{4}F_{\mu\nu}\Gamma^{\mu\nu}\Gamma^\rho\partial_\rho\epsilon - i\Gamma_\rho F^{\rho\mu}\partial_\mu\epsilon + i\Gamma^\mu\partial_\mu\epsilon'\right\} \tag{2.4}$$

which is linear in the spinor field Ψ.

To deform the theory, we add a fermionic bilinear to the ten dimensional action as

$$S_H = \frac{1}{g^2_{10}}\int d^{10}x\frac{i}{2}\,\mathrm{Tr}\,\Psi^T\Gamma^0 H\Psi$$

where H is a real chiral bispinor such that $H = -\Gamma_{11}H = H\Gamma_{11}$ and such that $\Gamma^0 H$ is anti-symmetric. Because of the above consistency conditions, the expansion of H in Γ matrices correspond just to a three form (plus its dual).[1]

Obviously, this term breaks supersymmetry (2.2) and (2.3) and our aim is to construct deformations of the field variations δ^0 and $\delta^{0'}$ and of the action such that no other fermionic term enters the action. A simple degree counting shows that this is possible by adding a further purely bosonic term S_b to the action if the new supersymmetry variation δ is such that

$$\delta A_\mu = \delta^0 A_\mu \qquad \text{and} \qquad \delta\Psi = \delta^0\Psi + K[A]\epsilon \tag{2.5}$$

where $K[A]$ is real chiral bispinor such that $\Gamma_{11}K[A] = K[A]\Gamma_{11} = K[A]$ which we take to be a local functional of the bosonic field A_μ only. We drop the condition that ϵ is constant. In terms of Γ-matrices analysis, $K[A]$ is given in terms of even self dual tensors. The closure of the the deformed susy algebra implies that $K[A]$ is linear[2] in A_μ.

We consider therefore the total action functional

$$S = S^0_{10} + S_H + \frac{1}{g^2_{10}}S_b \tag{2.6}$$

and we impose the invariance $\delta S = 0$ under the transformations (2.5). The total variation δS turns out to be linear in the fermion field Ψ and therefore we are left with a single condition

$$-\frac{1}{4}F^{\mu\nu}\Gamma_{\mu\nu}\Gamma^\rho\partial_\rho\epsilon - \Gamma_\rho F^{\rho\mu}\partial_\mu\epsilon - \frac{1}{4}HF^{\mu\nu}\Gamma_{\mu\nu}\epsilon + (\Gamma^\rho D_\rho + H)\,K[A]\epsilon - \frac{1}{2}\frac{\delta S_b}{\delta A_\mu}\Gamma_\mu\epsilon = 0 \tag{2.7}$$

equivalent to $\delta S = 0$.

[1] This is actually the standard analysis of type I R-R tensor fields.

[2] More general forms of $K[A]$ could perhaps be considered in an open superalgebra framework.

Notice that S is always invariant under $\delta' = \delta^{0'}$ if ϵ' satisfies the condition $(\Gamma^\mu \partial_\mu + H)\epsilon' = 0$. Further supersymmetries are then classified by the possible solutions of eq. (2.7).

Notice that if we would require ten dimensional Lorentz invariance, we would get $H = 0$ and therefore this deformation is naturally suitable for dimensionally reduced theories, where the Lorentz invariance along the reduced dimensions becomes R-symmetry which will be eventually broken[3] together with some of the original supersymmetries. In studing eq. (2.7), we will assume that $K[A]$ and S_b are respectively gauge covariant (a section of the adjoint gauge bundle) and gauge invariant with respect to the gauge group of the dimensionally reduced theory.

Equations analogous to (2.7) can be worked out for reductions of deformed lower dimensional N=1 SYM theories and/or for SYM theories coupled with matter.

3. Matrix theory on the pp-wave

Matrix theory [9] can be obtained by dimensional reduction of SYM_{10} along nine space dimensions. There has been recently [3, 7] proposed a model for matrix theory on pp-wave backgrounds in eleven dimension.

Here we show how the model there obtained follow as a particualr cases from dimensional reducion along nine space directions of the deformed ten dimensional SYM that we studied in the previous section.

In particular we show that all the set of supersumerary supergravity solutions studied in [7] are contained in a larger class of matrix models.

Let us split the ten dimensional index as $\mu = 0, I$, where $I = 1, \dots, 9$.

We parametrize $K[A] = K^I A_I$ and consider the following bosonic functional

$$S_b = \frac{1}{g^2} \int dx^0 \, \mathrm{Tr} \left\{ Q^{IJ} A_I D_0 A_J + M^{IJ} A_I A_J + N^{IJK} A_I \left[A_J, A_K\right] \right\}$$

where Q and N are completely antisymmetric and M is symmetric. Substituting S_b in the dimensionally reduced (to $1+0$) equations (2.7), we get three bispinorial equations, namely

$$\begin{aligned}
\frac{1}{4}\left[\Gamma_{IJ}, H\right] + \frac{1}{2}\left(\Gamma_I K_J - \Gamma_J K_I\right) + \frac{1}{4}\Gamma_{IJ} L - \frac{1}{2}\left(3N^{KIJ}\Gamma_K - Q^{IJ}\Gamma_0\right)\left(\frac{1+\Gamma_{11}}{2}\right) &= 0 \\
-\frac{1}{2}\Gamma^{0I}(H+L) - \frac{1}{2} H \Gamma^{0I} + \Gamma^0 K^I + Q^{IJ}\Gamma^J \left(\frac{1+\Gamma_{11}}{2}\right) &= 0 \\
\left[H K^I - \Gamma^0 K^I \Gamma^0 (H+L) - M^{IJ}\Gamma_J\right]\epsilon &= 0 \quad (3.1)
\end{aligned}$$

where we parametrized $(\Gamma^0 \partial_0 + H + L)\epsilon = 0$, with L a generic chiral bispinor.

The first two equations imply

$$K^I = -\frac{1}{2}\left(\frac{1}{12}\Gamma^I N_3 + \frac{1}{4} N_3 \Gamma^I + \Gamma^0 Q^{IK}\gamma^K\right)\left(\frac{1+\Gamma_{11}}{2}\right)$$

[3] Anyhow, the kind of breaking of the full ten dimensional Lorentz invariance induced by the presence of the bi-spinorial $\bar{\Psi} H \Psi$ term resembles a higher rank breaking dual to the one induced by non commutativity parameters.

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and

$$H = \frac{1}{4}\left(N_3 + \Gamma^0 Q_2\right)\left(\frac{1+\Gamma_{11}}{2}\right) \qquad \text{and} \quad L = -\frac{1}{3}N_3\left(\frac{1+\Gamma_{11}}{2}\right)$$

where $N_3 = N^{IJK}\Gamma^{IJK}$ and $Q_2 = Q^{IJ}\Gamma^{IJ}$.

The third equation (3.1) reduces to

$$\left\{\Gamma^I (N_3)^2 + 6N_3\Gamma^I N_3 + 9(N_3)^2\Gamma^I - 12\Gamma^0\left(3N_3 Q^{IK}\Gamma^K + Q^{IK}\Gamma^K N_3\right) + 144\cdot 2M^{IJ}\Gamma^J\right\}\epsilon = 0 \tag{3.2}$$

which generalizes the structure equation for supernumerary supersymmetries of 11 dimensional pp-wave solutions found in [7, eq.(20)] to matrix models with couplings to constant magnetic fields Q.

The dimensionally reduced action — obtained by defining $A_I = igX_I$ and $\Psi = g\Theta$ — reads

$$S_{mmm} = S_{mm} + S_m$$

where

$$S_{mm} = \int dx^0\, \mathrm{Tr}\left\{-\frac{1}{2}D_0X^ID_0X^I - \frac{g^2}{4}\left([X^I,X^J]\right)^2 + \frac{i}{2}\Theta^t D_0\Theta - \frac{g}{2}\bar{\Theta}\Gamma^I[X^I,\Theta]\right\}$$

is the usual matrix theory action and

$$S_m = \int dx^0\, \mathrm{Tr}\left\{\frac{i}{8}\bar{\Theta}\left(N_3 + \Gamma^0 Q_2\right)\Theta - Q^{IJ}X_ID_0X_J - M^{IJ}X_IX_J - igN^{IJK}X_I\left[X_J,X_K\right]\right\}$$

is the deformation part.

The whole action S_{mmm} is invariant under the following field transformations

$$\begin{aligned}
\delta A_0 &= \frac{ig}{2}\bar{\epsilon}\Gamma_0\Theta \qquad \delta X^I = \frac{1}{2}\bar{\epsilon}\Gamma^I\Theta \\
\delta\Theta &= \frac{i}{2}D_0X^I\Gamma^{0I}\epsilon + \frac{g}{4}[X^I,X^J]\Gamma^{IJ}\epsilon - \frac{1}{2}\left(\frac{1}{12}\Gamma^I N_3 + \frac{1}{4}N_3\Gamma^I + \Gamma^0 Q^{IK}\gamma^K\right)X^I\epsilon
\end{aligned} \tag{3.3}$$

if the ϵ spinor satisfies eq. (3.2) and

$$\left(\Gamma^0\partial_0 + \frac{1}{4}\left(-\frac{1}{3}N_3 + \Gamma^0 Q_2\right)\right)\epsilon = 0 \qquad \text{and} \qquad \left(\Gamma^0\partial_0 + \frac{1}{4}\left(N_3 + \Gamma^0 Q_2\right)\right)\epsilon' = 0$$

Therefore we obtained a generalization of the matrix models for pp-wave of [7] and have shown that all the classification of pp-wave vacua can be obtained directly from the matrix theory.

Let us notice that the constant magnetic field Q_{IJ} appearing in S_{mmm} cannot be in general re-absorbed in the other couplings by a change of variables. One can try to re-absorb it by a coordinate change to a co-rotating frame as

$$X^I \to \left(e^{-tQ}\right)^{IJ} X^J \qquad \text{and} \quad \Theta \to e^{-\frac{1}{4}tQ_2}\Theta$$

together with a mass matrix redefinition $M^{IJ} \to M^{IJ} - 2Q^{IK}Q^{KJ}$. It turns out however that in the co-rotating frame one finds a new 3-tensor and a new mass matrix which are

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generically time dependent because of the induced rotation. The time dependence of the new 3-tensor and mass matrix can be avoided only if they are invariant under the rotation generated by the anti-symmetric matrix Q.

Therefore we conclude that the model S_{mmm} effectively generalizes the one studied in [7] if $[N_3, Q_2] \neq 0$ or $QM - MQ \neq 0$ which hold in the generic case.

4. Matrix string theory on the pp-wave

Type IIA Matrix String Theory [10, 11] can be obtained by reducing the $\mathcal{N} = 1$ D=10 SYM theory with gauge group U(N) down to two dimensions [6]. MST is directly formulated for the IIA string (and the heterotic as well), while it is just indirectly linked to the type IIB string by T-duality. We will follow a constructive procedure by looking for possible supersymmetric deformations of the original model induced by the presence of a constant additional fermionic bilinear. This can be done just specifying the general equation obtained above in the appropriate dimensionally reduced framework.

We split the ten dimensional index $\mu = (\alpha, I)$, with $\alpha = 0, 9$ and $I = 1, \ldots, 8$ and solve the dimensionally reduced version of equation (2.7).

By imposing two dimensional Lorentz invariance and two dimensional gauge covariance, we find the most general cubic bosonic additional action functional to be

$$S_b = \int d^2x \, \mathrm{Tr} \left\{ F_{09} A_I v_I + \frac{1}{2} M_{IJ} A_I A_J + \frac{1}{3} N^{IJK} A_I [A_J, A_K] \right\}$$

We find the following four equations

$$\begin{aligned} \Gamma^\gamma \Gamma^{09} \partial_\gamma \epsilon + H \Gamma^{09} \epsilon + v_1 \epsilon &= 0 \\ -\frac{1}{2} \Gamma^\gamma \Gamma^{\alpha I} \partial_\gamma \epsilon + \Gamma^\alpha K^I \epsilon - \frac{1}{2} H \Gamma^{\alpha I} \epsilon + \frac{1}{2} v^I \epsilon^{\alpha\beta} \Gamma_\beta \epsilon &= 0 \\ -\frac{1}{4} \Gamma^\gamma \Gamma^{IJ} \partial_\gamma \epsilon + \frac{1}{2} \left(\Gamma^I K^J - \Gamma^J k^I \right) \epsilon - \frac{1}{4} H \Gamma^{IJ} \epsilon - \frac{1}{2} N^{IJK} \Gamma^K \epsilon &= 0 \\ \Gamma^\gamma K^I \partial_\gamma \epsilon + H K^I \epsilon - \frac{1}{2} M^{IJ} \Gamma^J \epsilon &= 0 \end{aligned} \tag{4.1}$$

The first three equations are equivalent to

$$K^I = -\frac{1}{2} \left(\frac{1}{12} N_3 \Gamma^I + \frac{1}{4} v_1 \Gamma^I \Gamma^{09} + v^I \Gamma^{09} \right) \qquad \text{and} \quad H = \frac{1}{12} N_3 - \frac{1}{4} v_1 \Gamma^{09}$$

and the following differential equation for the spinor ϵ

$$\partial_\alpha \epsilon + \frac{1}{2} \Gamma_\alpha \left(\frac{1}{12} N_3 + \frac{3}{4} v_1 \Gamma^{09} \right) \epsilon = 0 \tag{4.2}$$

while the last equation (4.1) is the mass/flux equation

$$\begin{aligned} &\left(\frac{1}{12} N_3 \Gamma^I - \frac{1}{4} v_1 \Gamma^I \Gamma^{09} - v^I \Gamma^{09} \right) \cdot \left(\frac{1}{12} N_3 + \frac{3}{4} v_1 \Gamma^{09} \right) + \\ &\quad + \left(\frac{1}{12} N_3 \Gamma^I - \frac{1}{4} v_1 \Gamma^{09} \right) \left(\frac{1}{12} N_3 + \frac{1}{4} v_1 \Gamma^I \Gamma^{09} + v^I \Gamma^{09} \right) + M^{IJ} \Gamma^J = 0 \end{aligned} \tag{4.3}$$

Moreover the integrability condition for (4.2), which is given by

$$\left(N_3 + 9v_1\Gamma^{09}\right)^T \left(N_3 + 9v_1\Gamma^{09}\right)\epsilon = 0 \tag{4.4}$$

has to be considered.

The reduced action then reads

$$S_{mms} = S_{ms} + S_m \tag{4.5}$$

where

$$\begin{aligned} S_{ms} = \int d^2x\,\mathrm{Tr}\Big\{ & -\frac{1}{4g^2}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}\eta^{\alpha\beta}D_\alpha X^I D_\beta X^I - \frac{g^2}{4}\left([X^I, X^J]\right)^2 + \\ & + \frac{i}{2}\bar{\Theta}\Gamma^\alpha D_\alpha\Theta - \frac{g}{2}\bar{\Theta}\Gamma^I[X^I, \Theta]\Big\} \end{aligned} \tag{4.6}$$

is the usual matrix string theory action and

$$S_m = \int d^2x\,\mathrm{Tr}\left\{\frac{i}{8}\bar{\Theta}\left(\frac{1}{3}N_3 - v_1\Gamma^{09}\right)\Theta + \frac{i}{g}F_{09}X_I v_I - \frac{1}{2}M_{IJ}X_I X_J - \frac{ig}{3}N^{IJK}X_I[X_J, X_K]\right\}$$

is the massive/flux part.

The action (4.5) is invariant under the following field variations

$$\begin{aligned} \delta A_\alpha &= \frac{ig}{2}\bar{\epsilon}\Gamma_\alpha\Theta \qquad \delta X^I = \frac{1}{2}\bar{\epsilon}\Gamma^I\Theta \\ \delta\Theta &= \Big(-\frac{1}{4g}F_{\alpha\beta}\Gamma^{\alpha\beta} + \frac{g}{4}[X^I, X^J]\Gamma^{IJ} - \frac{i}{2}D_\alpha X^I\Gamma^{\alpha I} - \\ & \quad - \frac{1}{2}\left(\frac{1}{12}N_3\Gamma^I + \frac{1}{4}v_1\Gamma^I\Gamma^{09} + v^I\Gamma^{09}\right)X^I\Big)\epsilon + \epsilon' \end{aligned} \tag{4.7}$$

provided eqs. (4.2), (4.3) and (4.4) hold for ϵ and ϵ' satisfies

$$\left(\Gamma^\alpha\partial_\alpha + \frac{1}{12}N_3 - \frac{1}{4}v_1\Gamma^{09}\right)\epsilon' = 0$$

Let us discuss the strong coupling limit as $g \to \infty$ of the above model. Considering the potential terms $g^2[X, X]^2$ and $g\Theta[X, \Theta]$, we find that at strong coupling all the fields are projected to a mutually commuting subset and the gauge group action becomes a parametrization of this choice. Once the choice of a Cartan subalgebra T is made, the leftover gauge group is generically $\mathrm{U}(1)^N$ semidirect with the Weyl group S_N.

At strong coupling the physical gauge fields degrees of freedom decouple from the other fields and the curvature field $F_{09}/g = F$ plays the role of an auxiliary field.[4] The action at strong coupling is therefore (all the fields are now Cartan valued)

$$\begin{aligned} S^\infty_{mms} = \int d^2x\,\mathrm{Tr}_T\Big\{ & +\frac{1}{2}F^2 + \frac{1}{2}\eta^{\alpha\beta}\partial_\alpha X^I\partial_\beta X^I + \frac{i}{2}\bar{\Theta}\Gamma^\alpha\partial_\alpha\Theta + \\ & + \frac{i}{8}\bar{\Theta}\left(\frac{1}{3}N_3 - v_1\Gamma^{09}\right)\Theta + iFX_I v_I - \frac{1}{2}M_{IJ}X_I X_J\Big\} \end{aligned} \tag{4.8}$$

[4] Notice that in two dimensions a gauge field has only a finite number of degrees of freedom [12] whose role in matrix string theory has been studied in [13]. We disregard these finite degrees of freedom for the time being.

and the action is invariant under the transformations

$$\delta F = \frac{i}{2}\left[\partial_0\left(\bar\epsilon\Gamma_9\Theta\right) - \partial_9\left(\bar\epsilon\Gamma_0\Theta\right)\right] \qquad \delta X^I = \frac{1}{2}\bar\epsilon\Gamma^I\Theta$$
$$\delta\Theta = -\frac{1}{2}\left(F\Gamma^{09} + \partial_\alpha X^I\Gamma^{\alpha I} + \left(\frac{1}{12}N_3\Gamma^I + \frac{1}{4}v_1\Gamma^I\Gamma^{09} + v^I\Gamma^{09}\right)X^I\right)\epsilon + \epsilon' \qquad (4.9)$$

under the same constrains as before.

Integrating out the auxiliary field F, we get a light-cone string action for the (matrix) strings given by

$$S = \frac{1}{2}\int d^2x Tr_T\Big\{\eta^{\alpha\beta}\partial_\alpha X^I\partial_\beta X^I + i\bar\Theta\Gamma^\alpha\partial_\alpha\Theta + \frac{i}{4}\bar\Theta\left(\frac{1}{3}N_3 - v_1\Gamma^{09}\right)\Theta -$$
$$- \left(M_{IJ} - v_I v_J\right)X_I X_J\Big\}$$

This result is obtained as strong coupling limit of the classical action and it should be checked if there are quantum corrections adding further complicancies. These are absent in the flat matrix string theory case because of strong non renormalization theorems for the theory with the full $(8,8)$ supersymmetry. It would be important to verify if these non renormalization properties are still effective for the models we have been considering here. The role of the abovementioned discrete gauge theory degrees of freedom should also be considered in detail.

4.1 A set of models

A particular set of Matrix String Theories in pp-wave backgrounds can be given for example as follows.

We solve the constraints (4.3) and (4.4) with the choice

$$N_3 = 9\mu\eta\Gamma^{123} \qquad \text{and} \qquad v_1 = \mu\Gamma^4$$

where μ is a mass parameter and $\eta = \pm 1$.

Moreover we break 1/4 of the original dymamical supersymmetry, by choosing

$$(\Gamma^{09} + \eta')\epsilon = 0 \qquad \text{and} \qquad (\Gamma^{1234} - \eta\eta')\epsilon = 0$$

where $\eta' = \pm 1$. The two flags η and η' label with their values four different models.

The mass matrix M^{IJ} turns out diagonal

$$M = \frac{\mu^2}{16}\left[\left(3\eta'(3+\eta) - 4\right)\mathbf{1}_3 \oplus 12(1+\eta')\mathbf{1}_1 \oplus 2(3\eta'+1)\mathbf{1}_4\right] \qquad (4.10)$$

and these models all break the original SO(8) R-symmetry of the flat ($\mu = 0$) model to $\mathrm{SO}(3) \times \mathrm{SO}(4)$. The full action is

$$S^{\eta\eta'}_{mms} = S_{ms} + S^{\eta\eta'}_{m}$$

where S_{ms} is the flat Matrix String action of the previous section and $S_m^{\eta\eta'}$ is given by

$$S_m^{\eta\eta'} = \int d^2x \,\mathrm{Tr}\Bigg\{ \frac{i\mu}{2}\bar\Theta\left[\frac{3}{4}\eta\Gamma^{123} - \frac{1}{4}\Gamma^{409}\right]\Theta + \frac{i\mu}{g}F_{09}X^4 - \frac{ig\mu\eta}{2}\sum_{I,J,K=1}^{3}\varepsilon^{IJK}X_I[X_J,X_K] - \\ - \frac{\mu^2}{32}\left[(3\eta'(3+\eta)-4)\sum_{I=1}^{3}(X^I)^2 + 12(1+\eta')(X^4)^2 + 2(3\eta'+1)\sum_{I=5}^{8}(X^I)^2\right]\Bigg\} \tag{4.11}$$

The deformed supersymmetry transformations are then

$$\begin{aligned} \delta A_\alpha &= \frac{ig}{2}\bar\epsilon\Gamma_\alpha\Theta \qquad \delta X^I = \frac{1}{2}\bar\epsilon\Gamma^I\Theta \\ \delta\Theta &= \Bigg(-\frac{1}{4g}F_{\alpha\beta}\Gamma^{\alpha\beta} + \frac{g}{4}[X^I,X^J]\Gamma^{IJ} - \frac{i}{2}D_\alpha X^I\Gamma^{\alpha I} - \\ &\quad - \frac{1}{2}\left(\frac{3}{4}\mu\eta\Gamma^{123}\Gamma^I + \frac{1}{4}\mu\Gamma^4\Gamma^I\Gamma^{09} + \mu\delta^{I4}\Gamma^{09}\right)X^I\Bigg)\epsilon + \epsilon' \end{aligned} \tag{4.12}$$

and the action is left invariant provided the above conditions on ϵ are satisfied and ϵ' satisfies

$$\left[\Gamma^\alpha\partial_\alpha + \frac{\mu}{4}\left(3\eta\Gamma^{123} - \Gamma^{409}\right)\right]\epsilon' = 0$$

5. Conclusions and open questions

In this letter we provided a set of possible models for Matrix String Theory on different pp-wave backgrounds and a general scheme to analyse their supersymmetries. It will be of interest to see how they will enter the gauge/string correspondence (a role played by matrix string theory has been recently advocated in [14]).

A crucial check of MST is the interpretation of the inverse gauge coupling as the actual type IIA string coupling. This was obtained in [11] by a carefull analysis of the supersymmetry preserving vacua. These are classified my solutions of an integrable system, the Hitchin equations, whose spectral surfaces materialize the interacting worldsheet diagrams in the strong coupling expansion. A proper index counting on these spectral surfaces provides in the relevant path integrals the crucial factor of $(1/g)^{-\chi}$, where χ is the Euler characteristic of the string worldsheet, and this implies the interpretation of $\frac{1}{g}$ as the string coupling g_s. Therefore, it is crucial to study susy-preserving configurations to check if and how the presence of the pp-wave background changes the reconstruction of the superstring interaction. This will hopefully provide a clear scheme — which is *intrinsic* the Matrix String Theory hypothesis — for the study of closed string interaction in a pp-wave background.

Let us notice that we can perform a full dimensional reduction to zero dimensions while keeping our scheme (at the price of excluding at least some kinematical supersymmetries) and get a deformed version of the IKKT matrix model [8]. This is done in appendix A. It would be nice to understand how to interpret this deformation from the type IIB string point of view of [8].

As it is evident from the results obtained here, the number of constrains on the surviving supersymmetries increases with the number of unreduced dimensions. It would be interesting to study also deformations of higher dimensional SYM theories along the paths of the general scheme presented here to see if some of them still can have some supernumerary supersymmetries.

Acknowledgments

I would like to thank M.Bertolini and L.Bonora for discussions. Work supported by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime in which G.B. is associated to Leuven.

A. Deforming the IKKT model

The deformation of the IKKT model [8] along the scheme outlined in this letter shows a peculiar phenomenon of a complete breaking of the kinematical supersymmetries ϵ'. In this appendix we present the formulas for the relevant deformation.

We perform dimensional reduction of the ten dimensional gauge theory down to zero dimensions and we choose the bosonic completition of the action to be

$$S_b = \mathrm{Tr}\left\{M^{\mu\nu}A_\mu A_\nu + N^{\mu\nu\rho}A_\mu[A_\nu, A_\rho]\right\}$$

and find the following two equations (we have $K[A] = K^\mu A_\mu$)

$$\begin{aligned} \left\{H\Gamma^{\mu\nu} - 2\left(\Gamma^\mu K^\nu - \Gamma^\nu K^\mu\right) + 6N^{\mu\nu\rho}\Gamma_\rho\right\}\epsilon &= 0 \\ \left\{HK^\mu - M^{\mu\nu}\Gamma_\nu\right\}\epsilon &= 0 \end{aligned} \tag{A.1}$$

The first one is identically solved by

$$K^\mu = -\frac{1}{16}\Gamma^\mu N_3 - \frac{1}{8}N_3\Gamma^\mu \qquad \text{and} \quad H = \frac{1}{4}N_3$$

where, as usual, $N_3 = N_{\mu\nu\rho}\Gamma^{\mu\nu\rho}$, while and the last one out of (A.1) is left as a mass/flux equation

$$\left[N_3\left(\Gamma^\mu N_3 + 2N_3\Gamma^\mu\right) + 4^3 M^{\mu\nu}\Gamma_\nu\right]\epsilon = 0 \tag{A.2}$$

The deformed IKKT matrix model is given by the action matrix function

$$S_m^{IKKT} = S^{IKKT} + S_m$$

where

$$S^{IKKT} = \mathrm{Tr}\left\{-\frac{g^2}{4}\left[X^\mu, X^\nu\right]^2 - \frac{g}{2}\bar{\Theta}\Gamma_\mu\left[X^\mu, \Theta\right]\right\}$$

is the IKKT matrix model for the flat IIB Schild action and

$$S_m = \mathrm{Tr}\left\{\frac{i}{8}\bar{\Theta}N_3\Theta - M^{\mu\nu}X_\mu X_\nu + igN^{\mu\nu\rho}X_\mu[X_\nu, X_\rho]\right\}$$

is the deformation part.

The total action is invariant under the transformations

$$\delta X^\mu = \frac{1}{2}\bar{\epsilon}\Gamma^\mu\Theta \qquad \text{and} \quad \delta\Theta = \frac{g}{4}\left[X^\mu, X^\nu\right]\Gamma_{\mu\nu}\epsilon - \frac{i}{16}X^\mu\left(\Gamma_\mu N_3 + 2N_3\Gamma_\mu\right)\epsilon \tag{A.3}$$

if (A.2) is satisfied. Notice the fact that all the kinematical linear supersymmetries ϵ' have been at least partly lost due to the constrain $N_3\epsilon' = 0$.

The easiest realization of these models can be obtained by studying the case $N_3 = \mu\Gamma^{789}$ for which, choosing the mass matrix to be

$$M = -\frac{\mu^2}{4^3}\left(\eta_7 \oplus 3\mathbf{1}_3\right)$$

we solve identically in ϵ the mass/flux constrain (A.2) and we retain the whole 16 ϵ supersymmetries — all the kinematical ones being lost — while breaking the ten dimensional Lorentz group SO(1,9) to SO(1,6) × SO(3).

References

[1] C.R. Nappi and E. Witten, *A WZW model based on a nonsemisimple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [hep-th/9310112].

[2] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[3] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[4] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. and Quant. Grav.* **19** (2002) L87–L95 [hep-th/0201081].

[5] U. Gursoy, C. Núñez and M. Schvellinger, *RG flows from* Spin*(7), CY 4-fold and HK manifolds to AdS, Penrose limits and pp waves*, *J. High Energy Phys.* **06** (2002) 015 [hep-th/0203124];
K. Oh and R. Tatar, *Orbifolds, Penrose limits and supersymmetry enhancement*, hep-th/0205067;
K. Dasgupta, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Matrix perturbation theory for M-theory on a pp-wave*, *J. High Energy Phys.* **05** (2002) 056 [hep-th/0205185];
O. Bergman, M.R. Gaberdiel and M.B. Green, *D-brane interactions in type-IIB plane-wave background*, hep-th/0205183;
N.R. Constable et al., *pp-wave string interactions from perturbative Yang-Mills theory*, *J. High Energy Phys.* **07** (2002) 017 [hep-th/0205089];
D.J. Gross, A. Mikhailov and R. Roiban, *Operators with large R charge in $N = 4$ Yang-Mills theory*, hep-th/0205066;
D. Berenstein and H. Nastase, *On lightcone string field theory from super Yang-Mills and holography*, hep-th/0205048;
C. Kristjansen, J. Plefka, G.W. Semenoff and M. Staudacher, *A new double-scaling limit of $N = 4$ super Yang-Mills theory and pp-wave strings*, hep-th/0205033;
I. Bakas and K. Sfetsos, *pp-waves and logarithmic conformal field theories*, hep-th/0205006;
V. Balasubramanian, M.-x. Huang, T.S. Levi and A. Naqvi, *Open strings from $N = 4$ super Yang-Mills*, hep-th/0204196;

JHEP08(2002)022

M. Alishahiha and M.M. Sheikh-Jabbari, *Strings in pp-waves and worldsheet deconstruction*, *Phys. Lett.* **B 538** (2002) 180 [hep-th/0204174];
S. Mukhi, M. Rangamani and E. Verlinde, *Strings from quivers, membranes from moose*, *J. High Energy Phys.* **05** (2002) 023 [hep-th/0204147];
K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, *J. High Energy Phys.* **06** (2002) 025 [hep-th/0204054];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051];
E. Kiritsis and B. Pioline, *Strings in homogeneous gravitational waves and null holography*, hep-th/0204004;
H. Lu and J.F. Vazquez-Poritz, *Penrose limits of non-standard brane intersections*, *Class. and Quant. Grav.* **19** (2002) 4059 [hep-th/0204001];
P. Lee and J.-w. Park, *Open strings in pp-wave background from defect conformal field theory*, hep-th/0203257;
J.P. Gauntlett and C.M. Hull, *pp-waves in 11-dimensions with extra supersymmetry*, *J. High Energy Phys.* **06** (2002) 013 [hep-th/0203255];
D. Berenstein, E. Gava, J.M. Maldacena, K.S. Narain and H. Nastase, *Open strings on plane waves and their Yang-Mills duals*, hep-th/0203249;
C.-S. Chu and P.-M. Ho, *Noncommutative D-brane and open string in pp-wave background with b-field*, *Nucl. Phys.* **B 636** (2002) 141 [hep-th/0203186];
S.R. Das, C. Gomez and S.-J. Rey, *Penrose limit, spontaneous symmetry breaking and holography in pp-wave background*, hep-th/0203164;
J. Michelson, *(Twisted) toroidal compactification of pp-waves*, hep-th/0203140;
E. Floratos and A. Kehagias, *Penrose limits of orbifolds and orientifolds*, *J. High Energy Phys.* **07** (2002) 031 [hep-th/0203134];
T. Takayanagi and S. Terashima, *Strings on orbifolded pp-waves*, *J. High Energy Phys.* **06** (2002) 036 [hep-th/0203093];
M. Billo and I. Pesando, *Boundary states for gs superstrings in an Hpp wave background*, *Phys. Lett.* **B 536** (2002) 121 [hep-th/0203028];
J.G. Russo and A.A. Tseytlin, *On solvable models of type-IIB superstring in NS-NS and RR plane wave backgrounds*, *J. High Energy Phys.* **04** (2002) 021 [hep-th/0202179];
N. Itzhaki, I.R. Klebanov and S. Mukhi, *pp wave limit and enhanced supersymmetry in gauge theories*, *J. High Energy Phys.* **03** (2002) 048 [hep-th/0202153];
R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[6] L. Brink, J.H. Schwarz and J. Scherk, *Supersymmetric Yang-Mills theories*, *Nucl. Phys.* **B 121** (1977) 77.

[7] M. Cvetič, H. Lu and C.N. Pope, *M-theory pp-waves, Penrose limits and supernumerary supersymmetries*, hep-th/0203229.

[8] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115].

[9] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[10] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025;
T. Banks and N. Seiberg, *Strings from matrices*, *Nucl. Phys.* **B 497** (1997) 41 [hep-th/9702187];

JHEP08(2002)022

R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[11] G. Bonelli, L. Bonora and F. Nesti, *Matrix string theory, 2D SYM instantons and affine Toda systems*, *Phys. Lett.* **B 435** (1998) 303 [hep-th/9805071]; *String interactions from matrix string theory*, *Nucl. Phys.* **B 538** (1999) 100 [hep-th/9807232];
G. Bonelli, L. Bonora, F. Nesti and A. Tomasiello, *Matrix string theory and its moduli space*, *Nucl. Phys.* **B 554** (1999) 103 [hep-th/9901093]; *Heterotic matrix string theory and Riemann surfaces*, *Nucl. Phys.* **B 564** (2000) 86 [hep-th/9905092].

[12] J.E. Hetrick and Y. Hosotani, *QED on a circle*, *Phys. Rev.* **D 38** (1988) 2621.

[13] L. Bonora and C.S. Chu, *On the string interpretation of M(atrix) theory*, *Phys. Lett.* **B 410** (1997) 142 [hep-th/9705137].

[14] R. Gopakumar, *String interactions in pp-waves*, hep-th/0205174.

JHEP08(2002)022